

# CHANGES ON THE GUIDE RAIL ARTICLES AND RAIL CALCULATIONS

# BROUGHT WITH TS EN 81-20 and TS EN 81-50

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## PRESENTATION

Dear Members of the Industry,

The EN 81-20 and EN 81-50 Standards, which have been worked on with great effort for many years, was published by TSE in October 2014, but the EN 81-1+A3 standard has remained effective for some time.

With the abolishment of old standard on September 1st, 2017, the conformity assessment of each elevator to be designed and approved for its project after this date shall be made according to EN 81- 20 and EN 81-50 standards.

Compiling the new condition about the guide rails, which is an important part of the elevator design, and for which the calculation procedures are fully described in the standards, would surely be up to ASRAY, the leading organization on the guide rails. We would like to present this work compiled by our friend Serdar Tavaslıoğlu to the industry. We would like to thank him, and present you this work wishing a safe, accident-free and profitable work life or you.

Regards,

SEFA TARGIT

## CHANGES TO ARTICLES ON RAILS AND RAIL CALCULATIONS THAT ARE BROUGHT WITH TS EN 81-20/50 STANDARD

TS EN 81-20 and TS EN 81-50 standards were the greatest amendments in the last two decades on the lift standards. These standards bring many changes to the installation of the elevators and to the design of the elevator components. Different new rules will be enforced on many points from the certifications of the products to the calculation of the products. We would like to introduce the changes about the rails, and present them with a practical example, brought with this new standard that shall be in effect for the periodical inspection and registration of elevators that are placed on the market in accordance with the elevator concept or application projects approved by the relevant licensing authority as of 1/9/2017. Thus, we thought that our industry may adapt more easily to the section on rails in the new standard.

With the new standard, many differences have occurred on many subjects. As a result, differences in the definition of rails and calculation principles have occurred, too. These changes may be examined under two different headings: definitions and calculations.

# A) NEW DEFINITIONS ON RAILS

In the previous standard, the requirement that the rails shall be machined or made of rolled steel was limited with some applications. The new standard eliminates these special requirements, specifies the rails for the car as rolled steel and steel with machined surface, and removes any other special requirements. This definition is given in article 5.7.1.2.

**"TS EN 81-20 5.7.1.2** Guide rails shall be made of rolled steel or their friction surfaces shall be machined."

The forces acting on the guide rails have differences in the new standard. Although there had been no change in horizontal forces, two new forces were defined for the vertical forces. These are described in the article below.

"**EN 81-20 Article 5.7.2.3.1** The following forces on guide rails shall be taken in account for calculation of permissible stresses and deflections of guide rails: a) horizontal forces from guide shoes due to:

1) masses of the car and its rated load, compensation means, travelling cables, etc. or the counterweight, balancing weight, taking into consideration their suspension points and dynamic impact factors, and

2) wind loads in case of lifts outside a building with partially enclosed well.

b) vertical forces from:

1) braking forces of safety gears and pawl devices fixed on guide rails;

2) auxiliary parts fixed on the guide rail;

3) weight of guide rail, and

4) push through forces of rail clips;

c) torques due to auxiliary equipment including dynamic impact factors.

Articles b).3) and b).4) describe newly defined forces, and we shall dwell on and examine these forces. While the fact that the weight of the guide rail specified in b).3) is defined amongst vertical forces changes the calculations for buckling, it also brings up the requirement to consider the calculations for buckling for the normal operation running and loading calculations. This force is defined as  $(M_g^*g_n)$ . Mg shall be taken as the mass (kg) of a line of the guide rails.  $F_p$  is described as push through forces of all brackets at one guide rail (due to normal settling of the building or shrinkage of concrete). For the rails secured from the top or the bottom, it may be formulated as  $F_p = n_b^*F_r$ . Here,  $F_p$  is described as the pushing force of all brackets available on a guide rail, and  $F_r$  is described as push through force of all clips per bracket. For rails that are not secured from any points (such as counter-weight rails), the formula is used as  $F_p = n_b^*F_r/3$ .

Besides the force  $F_p$  that is created by pushing of the rail brackets, another concept that is included in the calculations recently is the "combined deflection". The requirement to add the deflection caused by the structure of the building to the deflection caused by lateral forces is described in the article below.

"**TS EN 81-20 Article 5.7.2.1.2** The combination of deflections of guide rails and deflections of brackets, play in the guide shoes and straightness of the guide rails shall be taken into account in order to ensure a safe operation of the lift."

Thus, deflection calculations are formulated as follows:

 $\begin{array}{l} \delta_x = (0,7 \, . \, \mathsf{F}_X \, . \, \mathsf{L}^3) \, / \, (48 \, . \, \mathrm{E} \, . \, \mathsf{I}_Y) \, + \delta_{\text{str-x}} \, \leq \delta_{\text{perm}} \\ \delta_y = (0,7 \, . \, \mathsf{F}_Y \, . \, \mathsf{L}^3) \, / \, (48 \, . \, \mathrm{E} \, . \, \mathsf{I}_X) \, + \delta_{\text{str-y}} \, \leq \delta_{\text{perm}} \end{array}$ 

The recently added concepts in the formulation given in the standard are the  $F_p$  (pushing force of all brackets available in a guide rail) and the values of  $\delta_{str-x}$  (bending amount of the building structure at X axis (mm)) and  $\delta_{str-y}$  (bending amount of the building structure at Y axis (mm)). However, as described in Note to the Article 5.7.2.3.5 and Annex E 2, these values are factors that shall be taken into account for steel structures or similar structures with higher elasticity. Explanations on this subject are given below.

"*Article 57.2.3.5,* NOTE: F<sub>p</sub> depends on the way the guide rail is supported, the number of fixations, brackets and clip design. For small travels the effect of the settling of the building (not made of timber) is small and can be absorbed by the elasticity of the brackets. In this case the use of non sliding clips is of common practice.

For travel heights not exceeding 40 m the force  $F_p$  may be ignored in the formula. The design shall allow for adequate clearances above and/or below the guide rails depending on the fixation to allow for the shrinkage of the building.."

Also, Annex E 2 describes this subject as follows.

## "E.2 Supports of guide rails

When considering buildings constructed of concrete, blockwork or bricks it can be assumed that the guide rail brackets which support the guides will not be subjected to displacement caused by movement of the well walls (other than compression, see 5.7).

The total permissible deflection of the guide rails for the safe operation of the safety gear, etc, shall include any displacement of the guide rail due to deflection of the building fabric and the deflection of the guide it's self due to the load imparted on it by the car."

It is therefore important that the persons responsible for the design and fabrication of these supporting structures communicate with the lift provider in order to ensure that they are suitable under all load conditions.. "

As specified in the standard, the pushing forces that shall be caused by both the combined deflection and the rail brackets are forces that shall be notified by the builder of the building. The elasticity of the building is an important factor for very high buildings or for steel structures, however, for concrete buildings with lesser floors, these factors may be neglected. As a conclusion of the information provided above, you are not required to consider these forces for all buildings, excluding wooden buildings, up to a travel distance of 40 m, and for concrete buildings that are higher than the travel distance of 40 m, but that do not have much floors and for which no deflection is notified by its builder. However, these forces shall be included in the calculations when they are notified by the builder. (And the lift manufacturer shall ask the builder for this information, parties are mutually held responsible.)

A new approach is adopted for the calculation methods in the new standard. While the calculation method specified in the standard was held mandatory in the past, two new methods in addition to this method is adopted in the new standard. Article 5.7.4.7 provides an explanation of the calculation. Point b) describes that the method in EN 1993-1-1, also named as Eurocode 3, (EN 1993-1-1 Design of steel structures - Section 1-1: General rules and rules applicable for the buildings (Eurocode 3)) may be used, and option c) describes that the finite elements method (FEM) may also be used.

# "TS EN 81-20 Article 5.7.4.7 Calculation

Guide rails shall be calculated according to:

a) EN 81-50:2014, 5.10; or

b) EN 1993-1-1; or

c) Finite Element Method (FEM)."

With the addition of new forces to the calculation, the load chart has also changed. You shall decide what you shall take into account for the calculations as per these charts. These changes are described in Article 5.7.3.

## "TS EN 81-20 Article 5.7.3 Combination of loads and forces

Loads, forces and load conditions to be taken into account are given in Table 13."

Table 13 — Loads and forces to be taken into consideration in the different load cases									
Load conditions	Loads and forces	Ρ	Q	M <sub>cwt</sub> /M <sub>bwt</sub>	Fs	FP	Ma	Maux	WL
	Running	+	+	+		+ <sup>a</sup>	+	+	+
Normal operation	Loading + unloading	+	142	-	+	+ <sup>a</sup>	+ +	+	
Safety device		+	+	+	141	+ <sup>a</sup>	+	+	<b>1</b>
Operation									
		<sup>a</sup> Refer to article 5.7.2.3.5. (For elastic							
Note - loads and forces may not act simultaneously			buildings exceeding a travel distance of						

# **B) PRINCIPLE OF CALCULATION FOR THE GUIDE RAILS**

The standard provides a manual in EN 81-50 Standard for the rail calculations. Rail calculations include calculation of bending and buckling stresses that act on the rail, calculations of deflection, and performance of calculations defined as flange bending stress. Calculations that shall be made and the sizing of rails as per these calculations are described in the article below.

# "EN 81-50 Article 5.10.1 Range of calculation

Guide rails shall be dimensioned taking into account the following:

- Bending stress,
- Combined bending,
- Buckling stress,
- Compressive stress/tension stress,
- Combined bending and compression/tension stress,
- Combined bucling and bending,
- Flange bending stress,

In addition deflections shall be analysed "

Bending and buckling stresses may be calculated based on these assumptions. But, we shall discuss two values before moving on to these calculations. The first one is the assumptions to render the static loads to dynamic and the consideration of impact factor k. The second point is the determination of permissible stress values to decide whether the stresses are suitable.

## Determination of the factors used for rail calculations and allowed stress value

Before moving on to the principles of the calculation, we shall examine the assumptions to render the static loads to dynamic, the values of impact factor k, and safe stress values; a chart is provided in the standard for impact factor k. Values for factor k shall be taken as per braking and normal operating conditions from this chart. When the motor frames are fixed to the rails in MRL lifts, the upwards braking shall be taken into account and  $k_3$  dynamic factor shall be taken as the value of  $k_1$  if the upward brake is done from the car, and if the upwards braking of the lift is provided with electro-mechanic braking by the motor, a value shall be used by taking the bounce into account. Remember that buffered brakes are abolished, and instantaneous brakes or progressive brakes shall be used as mechanical brakes with the TS EN 81-20 standard.

Impact at	Impact factor	Value
Operation of instantaneous safety gear, not of the captive roller type		5
Operation of instantaneous safety gear, of the captive roller type or pawl device with energy accumulation type buffer or energy accumulation type buffer	k <sub>1</sub>	3
Operation of progressive safety gear or pawl device with energy dissipation type buffer or energy dissipation type buffer		2
Rupture valve		2
Running	k <sub>2</sub>	1.2
Auxiliary parts fixed to the guide rail and other operational scenarios	k <sub>3</sub>	(manufacturer)

## Table 14 — Impact factors

Permissible stress values, such as k impact factor values, that shall be used in the calculations to be made shall also be known. The criterion for assessment of the calculation results shall be the comparison that shall be made using these values. Permissible stress values to be assumed for the lifts are given in article 5.7.4.5.

"**TS EN 81-20 Article 5.7.4.5** Permissible stresses Permissible stresses shall be determined by:

 $\sigma_{perm} = R_m / S_t$ where;  $R_m \text{ is the tensile strength in N/mm^2,}$   $\sigma_{perm} \text{ is the permissible stress in N/mm^2,}$   $S_t \text{ is the safety factor.}^n$ 

These values correspond to  $R_m 370 \text{ N/mm}^2$  for cold drawn rails, and 440-520 N/mm<sup>2</sup> for machined rails when we refer to the steel charts. Nevertheless, "St" value of the rails used shall be taken from their manufacturer or certificates as different applications are possible. These materials used are considered as materials complying with A<sub>5</sub>>12% and their  $\sigma_{perm}$  values shall be taken as per the following table by paying attention to the yield strength and proportional strength.

			R <sub>m</sub>	
Load cases		<b>370</b> N/mm²	<b>440</b> N/mm²	<b>520</b> N/mm²
Safety device operation	σ <sub>perm</sub> N/mm <sup>2</sup> (1,8)	205	244	290
Normal operation running and loading/unloading	σ <sub>perm</sub> N/mm <sup>2</sup> (2,25)	165	195	230

After rendering static loads to dynamic and achieving the permissible stress value information to determine whether the stress values to be obtained are proper, you may calculate the stress values required by the standard and assess their conformity. Following the order provided by the standard, the bending stress shall be examined first.

# **Bending stress**

The  $\sigma_m$  bending stress shall be examined first when we follow this calculation order. Bending stress  $\sigma_m$  is the ratio of bending moment (M<sub>m</sub>) to the material's cross sectional area modulus (strength moment) (W). Some assumptions are made for the calculation of bending stress in the standard. Principles of calculation depend on these assumptions. You shall know these assumptions to understand some factors used in the formulas.

These assumptions are given in the article below.

"**TS EN 81-50 Article 5.10.2.1** Calculating the bending stresses in the different axis of the guide rail (Figure 4), it can be assumed that:

- The guide rail is a continuous beam with flexible fixing points at distances of the length l;
- The resultant of forces causing bending stresses act in the middle between adjacent fixing points;
- Bending moments act on the neutral axis of the profile of the guide rail."

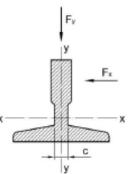


Figure 4 — Axis of the guide rail

The article for the calculation method of the bending stress for a continuous beam secured from one side with supports on distance L and acted from the center is given below.

# "TS EN 81-50 Article 5.10.2.1 .....

Evaluating the bending stress -  $\sigma_m$  - from horizontal forces acting at right angles to the axis of the profile, the following formulae shall be used:

 $\sigma_m = M_m / W$  $M_m = (1/2 F_h).(1/2 L_k).3/4 = (3. F_h. L_k) / 16$ 

# Where,

 $\sigma_m$  Bending stress (N/mm<sup>2</sup>))

*M<sub>m</sub>* Bending moment occurring on the rail (Nmm)

W the cross sectional area modulus (Strength moment) (mm<sup>3</sup>)

 $F_h$  the horizontal force applied to the guide rail by the guide shoes in the different load cases (N)

*L<sub>k</sub>* the maximum distance between guide brackets (mm)"

Horizontal force that shall create the bending moment shall be determined as per Fh operating mode. Moment branches as per different axes occur at different operating modes. Principles of calculation section shall provide the formulation of  $F_h$  force with respect to X and Y axes as per the operating modes. Moreover, following assumptions are made, too, in the standard for the bending calculations in practice.

**"EN 81-50 Art. 5.10.2.3** If more than two guide rails are used, an equal distribution of the forces between the guide rails may be assumed, provided their profiles are identical.

**EN 81-50 Art. 5.10.2.4** If more than one safety gear is used, acting on different guide rails, it can be assumed that the whole braking force is equally distributed between the safety gears.

**EN 81-50 Art. 5.10.2.5** In the case of vertical multiplex safety gears acting on the same guide rail, it shall be assumed, that the braking force of a guide rail is acting on one point."

## **Buckling stress**

As we have mentioned, "Jager method" also known as the "omega method" is used for rail buckling calculations. In this method, the force that shall create the moment, depending on " $\lambda$ " slenderness coefficient, shall be increased by the " $\omega$ " omega coefficient and a safety level shall be obtained as per the permissible stress value to be compared. Thus, an adequate safety level with the value of the " $\omega$ " coefficient determined as per the moment branch and radius of gyration of the material to be used is achieved. The page below describes, for the "**omega** method", how to calculate the " $\lambda$ " slenderness and how to determine the " $\omega$ " omega value. How to use the Omega method is described in the article below in the standard.

## "EN 81-50 Art. 5.10.3 Buckling

Determining the buckling stresses the "omega" -method shall be used with the following formulae:"

Buckling stress  $\sigma_k = [(F_{v+} k_3.M_{aux}). \varpi] / A^"$ 

To calculate the " $\omega$ " value used in this formula, we shall calculate the " $\lambda$ " slenderness first.

 $\lambda = L_k / i_{min}$   $L_k = L$  (Upper case L is used in the formula as lower case L may be confused with I)  $i_{min} = (I/A)^{1/2}$  formulae shall be used.

Where,

A is the cross sectional area of a guide rail mm<sup>2</sup>;

 $F_v$  is the vertical force on a guide rail of the car, counterweight or balancing weight N;

 $k_3$  is the impact factor;

 $\sigma_k$  is the buckling stress N/mm<sup>2</sup>;

 $\varpi$  is the omega value.

 $\lambda$  is the slenderness;

i is the minimum radius of gyration mm;

L is the maximum distance between guide brackets mm;

Lk is the buckling length mm."

As per the slenderness obtained, calculation method or " $\omega$ " charts are used to calculate the " $\omega$ " value. Formulation of the calculation method below is provided, and " $\omega$ " charts are annexed to the end of the brochure

For steels with a tensile strength of Rm = 370 N/mm<sup>2</sup>

 $20 \le \lambda \le 60$ :  $\omega = 0,00012920 \cdot \lambda^{1,89} + 1$ ;

 $60 < \lambda \le 85$ :  $\omega = 0,00004627 \cdot \lambda^{2.14} + 1$ ;

85 <  $\lambda \le 115$ :  $\omega = 0,00001711 \cdot \lambda^{2.35} + 1,04$ ;

115 <  $\lambda \le$  250:  $\omega$  = 0,00016887 ·  $\lambda^{2.00}$ 

For steels with a tensile strength of Rm =520 N/mm<sup>2</sup>

 $20 \le \lambda \le 50 \ \omega$  = 0,00008240  $\cdot \lambda^{2,06}$  + 1,021;

 $50 < \lambda \le 70 \ \omega = 0,00001895 \cdot \lambda^{2,41} + 1,05;$ 

70 <  $\lambda \le 89 \omega$  = 0,00002447 ·  $\lambda^{2,36}$  + 1,03;

89 <  $\lambda \le 250 \omega = 0,00025330 \cdot \lambda^{2,00}$ .

For steels with a tensile strength **(Rm)** between **370** N/mm<sup>2</sup> and **520** N/mm<sup>2</sup>, the "omega" values shall be calculated using the formula below. This calculation shall always be performed as materials with a strength of **440** N/mm<sup>2</sup> are used particularly for machined rails.

 $\varpi(\lambda) = [(\varpi_{52}-\varpi_{37}).(R_m - 370) / (520-370)] + \varpi_{37}$ 

After calculating the bending and buckling stresses, conformity shall be assessed as per  $\sigma_{\rm perm}$ 

## Combined bending and buckling stresses

After calculating the bending and buckling stresses, combined bending and compression stresses shall be calculated. It shall be pointed out that the buckling and bending stresses are added mathematically. These calculations are described in the following articles in the standard.

**"TS EN 81-50 Arc. 5.10.4** Combination of bending and compression/tension or buckling stresses The combined bending and compression/tension or buckling stresses shall be evaluated using the following formula::

Bending stresses	$\sigma = \sigma_m = \sigma_x + \sigma_y \leq \sigma_{perm}$
Bending and compression/tension	$\sigma = \sigma_m + (F_v + k_3 M_{yard_{imcl}})/A \leq \sigma_{perm}$
Bending and buckling	$\sigma = \sigma_k + 0.9 \sigma_m \le \sigma_{perm}$

where:

- A the cross sectional area of a guide rail (mm<sup>2</sup>),
- $F_v$  the vertical force on a guide rail of the car, counterweight or balancing weight (N),
- *k*<sub>3</sub> the impact factor,
- M<sub>aux</sub> the force in a guide rail due to auxiliary equipment (N),
- $\sigma$  the combined stress (N/mm<sup>2</sup>),
- $\sigma_k$  the buckling stress (N/mm<sup>2</sup>),
- $\sigma_m$  the bending stress (N/mm<sup>2</sup>),"
- $\sigma_{perm}$  the permissible stress (N/mm<sup>2</sup>),
- $\sigma_x$  the bending stress in the X-axis (N/mm<sup>2</sup>),
- $\sigma_y$  the bending stress in the Y-axis (N/mm<sup>2</sup>)."

While examining various operating modes, stresses caused by lateral forces at different axes shall be calculated and these values shall be added to calculate the combined stress. Then, it shall be assessed by considering the  $\sigma_{perm}$  together with the buckling stress. Another important calculation here is the calculation defined as flange bending stress.

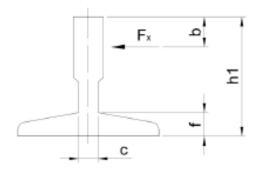
# Flange bending

As one of the resultant forces of the bending force that is composed of unbalanced loads, the force  $F_x$  also acts to bend the rail neck. The bending on the fishplates shall also be taken into account. The force that shall cause the bending on the rail neck "**c**" shall be  $F_x$ . Here, guide shoes with roller shoes and sliding shoes are differentiated and different calculation formulae are provided for these. The approach of the standard to this issue is defined in the article below.

# "TS EN 81-50 Arc. 5.10.5 Flange bending

Flange bending shall be taken into consideration. For T-shaped guide rails, the following formula shall be used::

for roller guide shoes  $\begin{aligned} \sigma_F = & (1,85 * F_x) / c^2 \leq \sigma_{perm} \\ \text{for sliding guide shoes} \\ \sigma_F = & (F_x * (h_1 - b - f) * 6) / (c^2 * (L + 2 * (h_1 - f)) \leq \sigma_{perm} \end{aligned}$ 



## Where;

- *b* the half the width of the guide shoe lining (*mm*),;
- c the width of the connecting part of the foot to the blade (mm),
- f the foot depth of guide rail at its connection with the blade (mm),
- *Fx* the force exerted by a guide shoe to the flange N;
- *h1* the guide rail height (*mm*),;
- *I* the length of the guide shoe lining *(mm)*,;
- $\sigma_F$  the local flange bending stress N/mm<sup>2</sup>;

 $\sigma_{perm}$  the permissible stress N/mm<sup>2</sup>

# **Deflections (Bending amount)**

Deflection on the rail shall be calculated as per the forces occurring on the rail. However, as the new standard requires that the deflection caused by the building shall also be considered, thus the combined deflection shall be considered, the new formula shall be as given below. The  $\delta_{str-x}$  and  $\delta_{str-y}$  values that shall be caused by the elasticity of the building shall also be considered and it shall be checked whether these values are within the values allowed by the standard. These values shall be provided by the builder of the building for buildings with high elasticity.

 $\delta_y \ = (0,7 \ . \ F_Y \ . \ L^3) \ / \ (48 \ . \ E \ . \ I_X) + \ \delta_{\ \text{str-y}} \qquad \text{on the y-y plane} \le \delta_{\text{perm}}$ 

$$\delta_x = (0,7 \cdot F_x \cdot L^3) / (48 \cdot E \cdot I_y) + \delta_{str-x} \text{ on the x-x plane} \le \delta_{perm}$$

 $\delta_{\text{perm}}$  the maximum permissible deflection (mm),

- $\overline{\delta_x}$  the deflection in the X-axis (mm),
- $\delta_y$  the deflection in the Y-axis (mm),
- $\delta_{\text{str-x}}$  the deflection of the building structure in the X-axis (mm),
- $\bar{\delta}_{str-y}$  the deflection of the building structure in the Y-axis (mm);
- E the modulus of elasticity (N/mm<sup>2</sup>)
- $F_x$  the supporting force in the X-axis (N),
- $F_y$  the supporting force in the Y-axis (N),
- $I_x$  the second moment of area in the X-axis (mm<sup>4</sup>), (Moment of inertia)
- $I_y$  the second moment of area in the Y-axis (mm<sup>4</sup>); (Moment of inertia)
- L the maximum distance between guide brackets (mm).

Please note that the  $F_x$  force shall be counter-acted by  $I_y$  moment of inertia, and the  $F_y$  force shall be counter-acted by the  $I_x$  moment of inertia. The permissible deflection amounts  $\delta_{perm}$  described in this article also have an important place in the lift calculations. This is a criterion that shall be taken into account while determining the sizes of rails. Deflection amounts permissible for lifts are described in article 5.7.4.6.

# "EN 81-20 Art.5.7.4.6 Permissible deflections

For T-profile guide rails and their fixings (brackets, separation beams) the maximum calculated permissible deflections  $\delta$ perm are:

a)  $\delta_{\text{perm}} = 5 \text{ mm}$  in both directions for car, counterweight or balancing weight guide rails on which Safety gears are operating;

b)  $\delta_{\text{perm}} = 10 \text{ mm}$  in both directions for guide rails of counterweight or balancing weight without safety gears.

Any deflection of building structure shall be taken into account in respect of guide rail displacement. See 0.4.2 negotiations and E.2."

# C) CALCULATIONS DEPENDING ON THE OPERATING CONDITIONS OF THE RAILS

Based on the calculation principles described above, calculations shall be performed as per various operating conditions on the lift. These operating conditions may be described as the following:

- **1.** Operation of the safety mechanism ( $\sigma_{perm}$  value shall be taken as per(1.8) S<sub>f</sub> Safety factor)
- **2.** Normal operation, running ( $\sigma_{perm}$  value shall be taken as per(2.25) S<sub>f</sub> Safety factor)
- **3.** Normal operation, Loading ( $\sigma_{perm}$  value shall be taken as per(2.25) S<sub>f</sub> Safety factor).

Each mode shall examined individually as they operate with different suspension axes, permissible stress values and forces. Operating conditions and calculations to be performed are given below.

## 1. Safety gear operation (Activation of the safety mechanism)

In this operating mode, while the safety gear is operated, " $\sigma_{perm}$ " values shall be assessed as per (1.8) **S**<sub>f</sub> Safety factor by taking the type of rail used. Value of the k<sub>1</sub> factor shall be selected considering the type of the mechanical brake used.

## a. Bending stress

The force  $F_h$  acting on the profile in the bending stress is a biaxial force. The stresses caused by the  $F_x$  and  $F_y$  forces acting vertically to the axes of the profile on both axes shall be calculated individually and bending stress shall be obtained as the sum of the stress on both axes. The formulae given in TS EN 81-50 Annex C Article C.2.1.1 may be used for the lateral  $F_h$  forces created here.

## "C.2.1.1 Bending stress

a) Bending stress relative to the Y-axis of the guide rail due to guiding force:

$$F_{x} = [k.g_{n}.(Q.x_{q}+P.x_{p})] / h.n$$
  

$$\sigma_{y} = M_{y} / W_{y} \qquad M_{y} = (3. F_{x}. L_{k}) / 16$$

b) Bending stress relative to the X-axis of the guide rail due to guiding force:

 $F_{y}=[k.g_{n}.(Q.y_{q}+P.y_{p})]/(h.n/2)$  $\sigma_{x}=M_{x}/W_{x} \qquad M_{x}=(3. F_{y}. L_{k})/16$ 

Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y \le \sigma_{perm}$ 

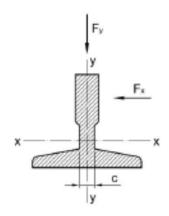


Figure 4 — Axis of the guide rail

- *F* Bending force in the axis
- g<sub>n</sub> acceleration of gravity
- Q Rated load
- P Total weight of the car

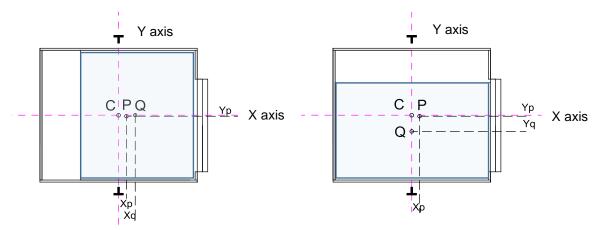
k impact coefficient depending on the operating condition and safety mechanism

 $x_q$  Length of the moment branch to the relevant point on the x axis of the rated load  $y_q$  Length of the moment branch to the relevant point on the y axis of the rated load

- $x_{\rho}$  Length of the moment branch to the relevant point on the x axis
- $y_p$  Length of the moment branch to the relevant point on the y axis
- h Distance between guide blades of the car
- n Number of the guide rails"

The direction of the  $F_x$  force used on the lift and the direction of the axis of the rail strength moment are not in the same direction. Thus, you shall pay attention that  $F_y$  force is used for calculating  $M_x$  and  $F_x$  force is used for calculating  $M_y$  as  $F_x$  force acts on the strength on Y axis of the rail.

<u>The values of  $x_p$ ,  $x_q$ ,  $y_p$ ,  $y_q$  shall be determined as per the center of the rail for the calculation of the bending stress of the safety mechanism.</u> Taking into account that rated load is distributed as unbalanced for a ratio of 75% on x and y axes, both conditions shall be examined, and the condition with the largest stresses shall be accepted. The uneven distribution the rated load when the safety mechanism is activated is shown below.



Case 1, load distribution branches for x axis

Case 2, load distribution branches for y axis

## b. Buckling stress

In safety gear operation, effects caused by auxiliary equipment, weight of the rails and pushing effect of the brackets shall be taken into account for the buckling stress.

Buckling stress  $\sigma_k = [(F_{v+k_3}.M_{aux}). \ \varpi] / A$ 

For the calculation of the buckling stress when the safety gear is activated, the  $F_v$  force defined as vertical in point b) for the car or the counterweight shall be calculated.  $F_v$  force is described in Article 5.7.2.3.5 of the standard.

" **EN 81-20 Atr. 5.7.2.3.5** The vertical force  $F_v$  of the car, counterweight or balancing weight resulting in compression or tension force shall be evaluated accordingly by using the formula:

For car	$F_v = [k_1.g_n.(P+Q) / n] + (M_g^*g_n) + F_p$
For counterweight	$F_v = [k_1.g_n. M_{cwt} / n] + (M_g^*g_n) + F_p$
For balancing weight	$F_v = [k_1.g_n. M_{cwt} / n] + (M_g^*g_n) + F_p$

in the case of guide rails supported on the pit or hanging, in case of freely hanging guide rails (no fixing point),  $F_p = n_b * F_r$  $F_p = n_b * F_r/3$ 

where

 $F_p$  the push through forces of all brackets at one guide rail

- $F_r$  the push through force of all clips per bracket N;
- $g_n$  the standard acceleration of free fall (9,81 m/s2);
- $k_1$  the impact factor according to Table 14

 $M_{g}$  the mass of one line of guide rails kg;

- *n* the number of guide rails;
- $n_b$  the number of brackets for a guide rail;

P the masses of the empty car and components supported by the car, i.e. part of the travelling cable, compensating ropes/chains (if any), etc. kg; Q is the rated load in kilograms.

 $(M_g*g_n)+F_p$  was explained before. If an auxiliary equipment (Maux) used by connecting to the rails are available in the system, it shall be added to the Fv force by multiplying it the dynamic effect factor k<sub>3</sub>. As we have discussed before, calculations for combined stress, flange bending stress and deflection are performed by considering the  $\sigma_{perm}$  value.

## c. Combined stres

Bending stresses Bending and compressive/tensile stress Bending and buckling stresses 
$$\begin{split} \sigma = &\sigma_m = \sigma_x + \sigma_y &\leq \sigma_{perm} \\ \sigma = &\sigma_m + (F_v + k_3.M_{aux})/A &\leq \sigma_{perm} \\ \sigma = &\sigma_k + 0.9\sigma_m &\leq \sigma_{perm} \end{split}$$

## d. Flange bending stress

 $\begin{array}{ll} \mbox{For roller type guide shoes} & \sigma_F = (1,85 \ ^* F_x) \ / \ c^2 \ \le \ \sigma_{perm} \\ \mbox{For sliding guide shoes} & \sigma_F = (F_x \ (h_1 - b - f) \ 6) \ / \ (c^{2*}(L + 2^*(h_1 - f)) \ \le \ \sigma_{perm} \\ \end{array}$ 

#### e. Deflections

on the y-y plane	$\delta_y = (0,7 . F_Y . L^3) / (48 . E . I_X) + \delta_{str-y}$	$\leq \delta_{\text{perm}}$
on the x-x plane	$\delta_x = (0,7 . F_x . L^3) / (48 . E . I_y) + \delta_{str-x}$	$\leq \delta_{\text{perm}}$

## 2. Normal operation, Running

In this operating mode,  $\sigma_{perm}$  values shall be assessed as per (2.25) S<sub>f</sub> Safety factor by taking the type of rail used. As a dynamic conversion factor, k<sub>2</sub> factor shall be determined by taking the effect of the electromechanical braking into account. This value may be taken as 1.2 if no special condition is present. The values of x<sub>p</sub>, x<sub>q</sub>, y<sub>p</sub>, y<sub>q</sub> shall be determined as per the suspension center of the car.

## a. Bending stress

In this case,  $F_x$  and  $F_y$  shall be formulated as specified in EN 81-50 Annex C Article C.2.2.1.

#### "EN 81-50 art. C.2.2.1 Bending stress

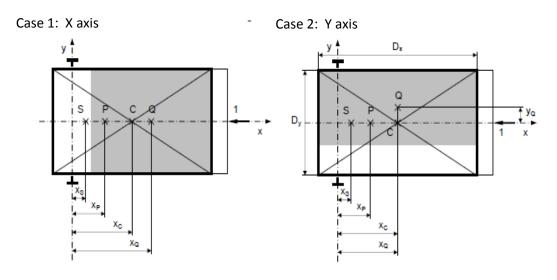
a) Bending stress relative to the Y-axis of the guide rail due to guiding force:

 $\begin{aligned} F_{X} &= k_{2}*g_{n}*[Q^{*}(x_{q}-x_{s})+P^{*}(x_{p}-x_{s})] \ / \ n^{*}h \\ \sigma_{y} &= M_{y} \ / \ W_{y} \qquad M_{y} &= (3. \ F_{x}. \ L_{k}) \ / \ 16 \end{aligned}$ 

b) Bending stress relative to the X-axis of the guide rail due to guiding force:  $F_{Y}=k_{2}\cdot g_{n}*[Q^{*}(y_{q}-y_{s})+P^{*}(y_{p}-y_{s})]/(h^{*}n/2)$  $\sigma_{x}=M_{x}/W_{x}$   $M_{x}=(3. F_{y}. L_{k})/16$  "

Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y \le \sigma_{perm}$ 

Unfavourable conditions of load distribution shall also be taken into account as per X and Y axes.



## **b. Buckling stress**

Effects caused by auxiliary equipment, weight of the rails and pushing effect of the brackets shall be taken into account for the buckling stress. On the previous calculations, we does not calculate buckling under normal operation running and loading conditions. But now, with new standard buckling calculations shall be performed for each operating mode with the inclusion of the rail weights to the buckling calculations. Nevertheless, pay attention that for the normal operation mode, the " $\omega$ " value is not used in the stress formulae. As the safety mechanism does not operate, rail weight and Fp forces are examined only.

$$F_v = (M_g^*g_n) + F_p$$
  
$$\sigma_k = (F_{v+} k_3.M_{aux})/A$$

#### c. Combined stress

Bending stresses Bending and compressive/tensile stress  $\sigma = \sigma_m + (F_v + k_3.M_{aux})/A \le \sigma_{perm}$ 

 $\sigma = \sigma_m = \sigma_x + \sigma_y \leq \sigma_{perm}$ 

#### d. Flange bending stress

For roller type guide shoes	$\sigma_F = (1,85 * F_x) / c^2 \le \sigma_{perm}$
For sliding guide shoes	$\sigma_{\text{F}} = (F_x^*(h_1\text{-}b\text{-}f)^*6)/(c^{2*}(L\text{+}2^*(h_1\text{-}f)) \le \sigma_{\text{perm}}$

#### e. Deflections

on the y-y plane  $\delta_y = (0,7 \cdot F_Y \cdot L^3) / (48 \cdot E \cdot I_X) + \delta_{str-y} \le \delta_{perm}$ on the x-x plane  $\delta_x = (0,7 \cdot F_x \cdot L^3) / (48 \cdot E \cdot I_y) + \delta_{str-x} \leq \delta_{perm}$ 

#### 3. Normal operation, Loading

In this operating mode,  $\sigma_{perm}$  values shall be assessed as per (2.25) Sf Safety factor by taking the type of rail used. Effects caused by auxiliary equipment, weight of the rails and pushing effect of the brackets shall be taken into account for the buckling stress. As the lift is not moving, k<sub>2</sub> factor shall not be taken into account. For the bending stress, instead of the Q value, a threshold force Fs that act on the center of the sill on the entrance of the car shall be taken into account while loading and unloading the car. The magnitude of the sill force shall be taken as specified in 5.7.2.3.6.

" EN 81-20 art. 5.7.2.3.6 Whilst loading or unloading a car, a vertical force on the sill Fs has to be assumed to act centrally on the sill of the car entrance. The amount of the force applied on the sill shall be:

- for passenger lifts;:  $F_{\rm S} = 0, 4.g_n .Q$ — for goods passenger lifts:  $F_{\rm S} = 0.6.q_n .Q$ - for goods passenger lifts in the case of heavy handling devices if the weight of the device is not included in the rated load.:  $F_{\rm S} = 0.85.q_n .Q$ 

When applying the force on the sill the car shall be regarded as empty. At cars with more than one entrance the force on the sill needs to be applied at the most unfavourable entrance only .. "

Calculations given below shall be performed

#### a. Bending stress

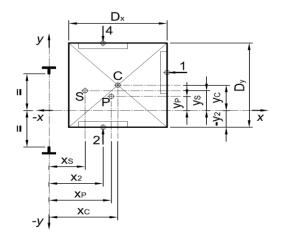
The car shall be assumed as empty when a force is applied on the sill. For cars with more than one entrances, it shall be assumed that the force is applied on the most unfavourable entrance only. In this case, Fx and Fy shall be formulated as specified in EN 81-50 Annex C Article C.2.3.1

## "EN 81-50 Art.C.2.3.1 Bending stress

- a) Bending stress relative to the Y-axis of the guide rail due to guiding force:  $F_{X}=[g_{n}*P^{*}(x_{P}-x_{s})+F_{S}*(x_{i}-x_{S})]/n*h$   $M_{y}=(3. F_{x}. L_{k})/16$   $\sigma_{y}=M_{y}/W_{y}$
- b) Bending stress relative to the X-axis of the guide rail due to guiding force:

 $F_{Y} = [g_n * P^*(y_{P}-y_s) + F_S^*(y_i-y_s] / (h*n/2)$  $M_x = (3. F_y. L_k) / 16 "$  $\sigma_x = M_x / W_x$ 

Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y \le \sigma_{perm}$ 



#### b. buckling stress

Effects caused by auxiliary equipment, weight of the rails and pushing effect of the brackets shall be taken into account for the buckling stress.

$$F_v = (M_g^*g_n) + F_p$$
  
$$\sigma_k = (F_{v+} k_3.M_{aux})/A$$

#### c. Combined stress

#### d. Flange bending stress

For roller type guide shoes	$\sigma_F = (1,85 * F_x) / c^2 \le \sigma_{perm}$
For sliding guide shoes	$\sigma_{F} = (F_x^{*}(h_1-b-f)^{*}6)/(c^{2*}(L+2^{*}(h_1-f)) \le \sigma_{perm}$

#### e. Deflections

on the y-y plane	$\delta_y = (0,7 . F_y . L^3) / (48 . E . I_X) + \delta_{str-y}$	$\leq \delta_{\text{perm}}$
on the x-x plane	$\delta_x = (0,7 . F_x . L^3) / (48 . E . I_Y) + \delta_{str-x}$	$\leq \delta_{\text{perm}}$

**NOTE =** (The same " $\sigma_{perm}$ " permissible stress values are taken under (Normal operation Running) and (Normal operation Loading) conditions. Thus, for these operation modes it may be sufficient to perform calculations for Buckling and Flange bending once by taking the maximum  $F_x$  and  $F_y$  values for these modes and verify these calculations. Also for the calculations of deflections, for the max  $F_x$  and  $F_y$  force for all modes, it may be calculated at the first mode. Because deflection does not depend on the " $\sigma_{perm}$ " value and the max forces generally occur on safety gear operations. After calculating for this mode, to compare the other mode forces have lower values than this mode, will be sufficient for safety. It is obvious that if the results are sufficient for the larger forces it will verify it for smaller forces. This is my opinion also, I suggest you do the all calculations.)

## **D) CALCULATIONS FOR COUNTERWEIGHT RAILS**

Besides the calculations for cars, calculations for counterweights shall also be performed. The assumptions in the standard for counterweight or balancing weight are specified in the following article

## " EN 81-20 Art. 5.7.2.3.3

The guiding forces of a counterweight *Mcwt* or balancing weight *Mbwt* shall be evaluated taking into account:

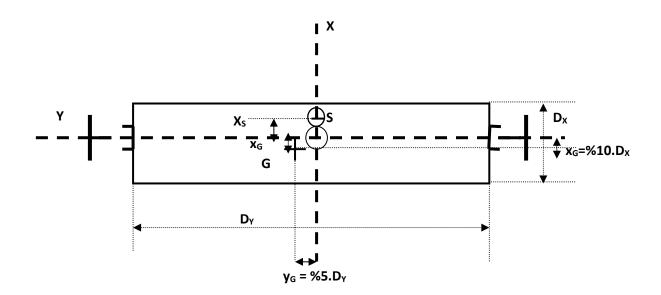
- the acting point of the mass;
- the suspension; and

- the forces due to compensating ropes/chains (if any), tensioned or not.

On a counterweight or balancing weight, centrally guided and suspended, an eccentricity of the acting point of the mass from the centre of gravity of the horizontal cross area of the counterweight or balancing weight of at least 5 % of the width and 10 % of the depth shall be taken into consideration.."

Rail calculations shall be performed as per the assumptions for the eccentricity of weight mass that shall be considered as the mass G. Although these forces are small on rail systems without a safety gear mechanism that do not carry any weight on them, these calculations shall be important particularly for rails of the MRL system that carries equipment and motor frames on them. Selecting rail cross-sections without making these calculations for the drive systems fixed to the rails and having them carried by the rails may pose great risks.

For MRL lifts,( $k_3.M_{aux}$ ) value shall be taken as [ $k_1.g_n.(P+Q+Machineweight+rope) / n$ ] when considering the upwards braking made from the car. N is the number of rails carrying the weight of the machine frame. Calculations for the counterweight rails shall be performed as follows. (The suspension point "S" is taken as eccentric in the x axis for explanatory purposes in the following example.)



- G Weight acting point of the mass of the counterweight or balancing ,
- **S** Suspension point of the counterweight or balancing weight
- $D_X$  Depth of the counterweight or balancing weight
- $D_Y$  Width of the counterweight or balancing weight
- $Y_G$  Distance of the center of gravity to the Y axis
- X<sub>G</sub> Distance of the center of gravity to the X axis
- Xs Distance of the suspension point to the X axis

## 1. Safety gear operation

#### **Bending stress**

 $F_{X} = (k_{1.}g_{n.}G. x_{G}) / n.h$   $M_{Y}= 3.F_{X.}L / 16$   $\sigma_{Y} = M_{Y} / W_{Y}$   $F_{Y} = (k_{1.}g_{n.}G. y_{G}) / n.h/2$   $M_{X}= 3.F_{Y.}L / 16$  $\sigma_{X} = M_{X} / W_{X}$ 

Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y \leq \sigma_{perm}$ 

## **Buckling stress**

For balancing or counterweights	$F_v = [k_1.g_n. M_{cwt} / n] + (M_g^*g_n) + F_p$
For the rails secured from the top or the bottom	$F_p = n_b * F_r$
For the rails that are not secured at any point	$F_p = n_b * F_r / 3$

 $\sigma_k = [(F_{v+} k_3.M_{aux}). \varpi] / A$ 

**Buckling stress** 

#### c. Combined stress

## d. Flange bending stress

 $\begin{array}{ll} \mbox{For roller type guide shoes} & \sigma_{\rm F} = (1,85 \ ^{*} \ F_x) \ / \ c^2 \ \leq \sigma_{\rm perm} \\ \mbox{For sliding guide shoes} & \sigma_{\rm F} = (F_x \ (h_1 - b - f) \ ^{*} b) \ / \ (c^{2*} (L + 2^* (h_1 - f)) \ \leq \sigma_{\rm perm} \\ \end{array}$ 

#### e. Deflections

 $\begin{array}{ll} \delta_y \ = (0,7 \ . \ F_Y \ . \ L^3) \ / \ (48 \ . \ E \ . \ I_X) + \ \delta_{\ str-y} & on \ the \ y-y \ plane \le \delta_{perm} \\ \delta_x \ = (0,7 \ . \ F_X \ . \ L^3) \ / \ (48 \ . \ E \ . \ I_Y) \ + \ \delta_{\ str-x} & on \ the \ x-x \ plane \le \delta_{perm} \end{array}$ 

## 2. Normal operation, running

Point **S** shall be taken as the axis of the moment branch in this case, and the calculations shall be performed as per this condition. Normal operation, Loading calculations shall not be performed.

#### **Bending stress**

 $\begin{array}{l} F_{X} = \left[k_{2}.g_{n}.G.\left(x_{G}+x_{S}\right)\right] / n.h \\ M_{Y} = 3.F_{X}.L \ / 16 \\ \sigma_{Y} = M_{Y} \ / W_{Y} \\ \mbox{If the point S is on the side of the point G, } (x_{G}-x_{S}) \ shall \ be \ used \ in \ the \ formula. \end{array}$ 

 $\begin{array}{l} F_{Y}=(k_{2}.g_{n}.G.\;y_{G})\;2\;/\;n.h\\ M_{X}=\;3.F_{Y}.L\;/16\\ \sigma_{X}=\;M_{X}\;/\;W_{X}\\ \mbox{If the point $S$ was eccentric on the $y$ axis, $(y_{G}+y_{s})$ shall be used in the formula} \end{array}$ 

Total bending stress

$$\sigma = \sigma_m = \sigma_x + \sigma_y \leq \sigma_{perm}$$

#### **Buckling stress**

 $F_v = (M_g^*g_n) + F_p$  $\sigma_k = (F_{v+} k_3.M_{aux})/A$ 

#### c. Combined stress

Bending stresses  $\sigma = \sigma_m = \sigma_x + \sigma_y \le \sigma_{perm}$ Bending and compressive/tensile stresses  $\sigma = \sigma_m + (F_v + k_3.M_{aux})/A \le \sigma_{perm}$ 

#### d. Flange bending stress

For roller type guide shoes	$\sigma_{F} = (1,85 * F_{x}) / c^{2} \leq \sigma_{perm}$
For sliding guide shoes	$\sigma_{F} = (F_x^*(h_1-b-f)^*6)/(c^{2*}(L+2^*(h_1-f)) \le \sigma_{perm}$

#### e. Deflections

on the y-y plane	$\delta_y = (0,7 . F_Y . L^3) / (48 . E . I_x) + \delta_{str-y}$	$\leq \delta_{\text{perm}}$
on the x-x plane	$\delta_x = (0,7 . F_x . L^3) / (48 . E . I_y) + \delta_{str-x}$	$\leq \delta_{\text{perm}}$

#### 3) If a safety gear is not used

If a safety mechanism is not used on the counterweight, the normal operation-movement calculations provided above shall be performed only. The only difference is on the inspection of the deflection. In this case, the deflection on the rail shall be smaller than 10 mm.

#### Deflections

 $\begin{array}{ll} \delta_X = (0,7.F_X.L^3) \: / \: (48. \ \mathrm{E.I_Y}) & < \delta_{perm} = 10 \ mm \\ \delta_Y = (0,7.F_Y.L^3) \: / \: (48. \ \mathrm{E.I_X}) & < \delta_{perm} = 10 \ mm \end{array}$ 

## REFERENCES

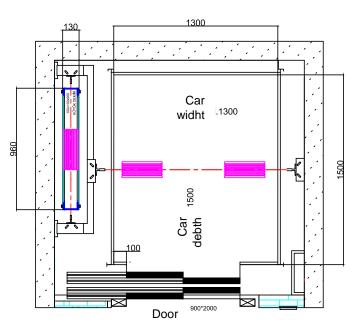
- 1. Elevator Design and Aplications Serdar Tavaslıoğlu Final yayınları November 2005
- 2. EN 81-20 Safety rules for the construction and installation of lifts Lifts for the transport of persons and goods Part 20: Passenger and goods passenger lifts
- 3. TS EN 81-50 Safety rules for the construction and installation of lifts Examinations and tests Part 50: Design rules, calculations, examinations and tests of lift components
- 4. TS ISO 7465 Passenger lifts and service lifts Guide rails for lift cars and counterweights Ttype
- 5. TS EN 81-1 + A3 Safety rules for the construction and installation of lifts Part 1: Electric lifts

#### DISCLAIMER

This manual is intended as a means for providing help for guidance of lifts and selection of guide rails within the cover of TS EN 81-20 and TS EN 81-50 standards. This manual is not intended for replacing the will of the lift technician designing the lift for their own examinations and assessments and making their own decisions. Asray and Serdar Tavaslıoğlu, as the compiler of this document, declares that they shall not accept any responsibility for measures that are taken or not taken based on this manual.

# EXAMINATION OF THE RAIL STRESSES ON DIFFERENT OPERATING CONDITIONS FOR 800 KG MRL LIFTS

The rail calculations for a 1 m/s passenger lift with a rated load of 800 kg, with a travelling distance of 40 m, and without machine room (MRL) for which the shaft cross-section is drawn and specifications are provided are performed below.



Q = 800 kg Rated load

P = 1000 Kg Car and sling weight,

K = 100 Kg Door weight,

Dx =1500 mm,

Dy =1300 mm

Dx/8=187.5mm,

Dy/8 =162.5 mm Car dimensions,

G = 1500 Kg Weight of Counterweight, (1000(P)+100(D)+400(Q/2))

Gx=130mm,

Gy =960 mm

 $X_G = 10\%$  of Gx=13mm,

 $Y_G = 5\%$  of Gy=48 mm Counterweight dimensions

Travelling distance = 40 m  $k_1$ = 2 Dynamic effect factor for sliding brake,  $k_2$  =1.2, L= 2500 mm The distance between the brackets,  $h_k$ = 3300 mm The distance between the car guide shoes,  $h_G$ =3000 mm The distance between the counterweight guide shoes, n=2 Double rail system,

δstr-x, δstr-y and Fp are not taken into account as the travelling distance is small.

Before moving on to the calculation for the MRL lift with sliding guide shoes, of which the rails are installed symmetric and suspended from the center, with a door as central panel, but placed eccentrically, and where T90 A rails are used for the car, and T70 A rails are used for the counterweight, the required parameters shall be obtained.

# A) CALCULATION PARAMETERS REQUIRED

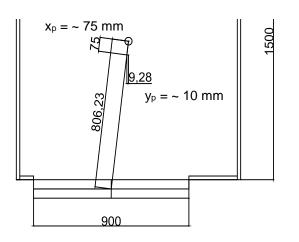
## a. Center of gravity of the car

 $x_{P}^{*}(Car weight kg) = (D_{x}-x_{P})^{*}(door weight kg)$ 

x<sub>P</sub>.1000 = (806 -x<sub>P</sub>).100 1000 x<sub>P</sub> = 80600-100 x<sub>P</sub> 1100 x<sub>P</sub> =80600

 $x_{\rm P}$  =80600/1100 = 73.27 mm, it shall be taken as  $\sim$  75 mm.

 $y_p = \sim 10$  mm. The value taken from the drawing as the door is placed eccentrically.



Distance between the centers of gravity = 806 mm



1100 kg center of gravity

## b. Description of the rails used

Rail	S	<b>q</b> 1	h1	С	f	Ixx	Wxx	İxx	Iyy	WYY	İyy
specification	cm <sup>2</sup>	kg/m	mm	mm	mm	cm4	cm <sup>3</sup>	cm	cm <sup>4</sup>	cm <sup>3</sup>	cm
T70 A	9,400	7,379	65	6	8	40,95	9,169	2,087	18,86	5,389	1,417
T90 A	17,25	13,54	75	10	10	102,00	20,86	2,431	52,48	11,66	1,744

#### c. Determination of the slenderness and omega

#### $\lambda = L_k / i_{min}$ $L_k = L$

 $i_{min}$  Radius of gyration must be used as the small value of  $i_{xx}$  or  $i_{yy}$  in mm.

Value of slenderness  $\lambda = 2500/17,44 = 143,34$ 

For A-type cold drawn rail of 370 N/mm<sup>2</sup>,

For  $115 < \lambda \le 250$   $\omega = 0,00016887 \cdot \lambda^{2.00}$ 

 $\omega$  = 3,46 (from the chart, for  $\lambda$ =143  $\omega$  =3,45, for  $\lambda$ =144  $\omega$  =3,50 shall be obtained)

## **B) CALCULATIONS FOR CAR RAILS**

#### 1. Safety gear operation

For a material with R<sub>m</sub>=370 N/mm<sup>2</sup>, with respect to the (1.8) Sf Safety factor,  $\delta_{perm}$ = 205

Value of the k1 impact factor = 2 for the progressive safety gear

The rated load Q of the car shall be evenly distributed over those three quarters of the car area

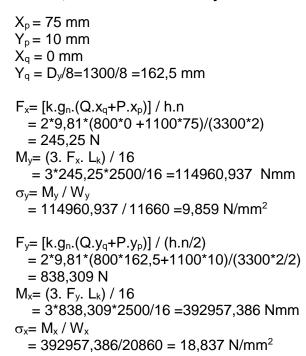
#### a. Bending Stress

#### Case 1, load distribution for x axis

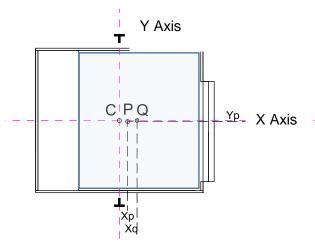
 $X_{p} = 75 \text{ mm}$  $Y_{p} = 10 \text{ mm}$  $X_q = D_x/8 = 187,5 \text{ mm}$  $Y_a = 0 mm$  $F_{x} = [k.g_{n}.(Q.x_{q}+P.x_{p})] / h.n$  $= 2^{9},81^{(800^{1}87,5)} + 1100^{7},5)/(3300^{2})$ = 691,159N  $M_v = (3. F_x. L_k) / 16$ = 3\*691,159\*2500/16 = 323980,78 Nmm  $\sigma_y = M_y / W_y$ = 323980,78 / 11660 = 27,929 N/mm<sup>2</sup>  $F_{y} = [k.g_{n}.(Q.y_{q}+P.y_{p})] / (h.n/2)$  $= 2^{9},81^{(800^{0}+1100^{10})/(3300^{2}/2)}$ = 65,4 N  $M_x = (3. F_y. L_k) / 16$ = 3\*65,4\*2500/16 = 30656,25 Nmm  $\sigma_x = M_x / W_x$ = 30656,25 / 20860 =1,469 N/mm<sup>2</sup>

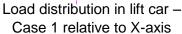
Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y = 27,929+1,469$  $\sigma = \sigma_m = 29,398 \text{ N/mm}^2$ 

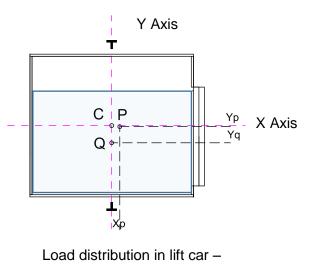
#### Case 2, load distribution for y axis



Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y = 9,859+18,837$  $\sigma = \sigma_m = 28,696 \text{ N/mm}^2$ 







Case 2 relative to Y-axis

It shall be observed that the stress on Case 1 is higher when we compare total bending stresses. This stress shall be taken as the worse condition for the calculations.

 $\sigma = \sigma_m = 29,398 \ N/mm^2$ 

#### b. Buckling stress

 $\begin{aligned} F_v &= [k_{1.}g_{n.}(P+Q) / n] + (M_g^*g_n) + F_p \\ M_g^*g_n &= 13,54^*40^*9,81 = 5313,09 \text{ N} \\ F_p &= 0 \\ F_v &= 2^*9,81^*(1100+800)/2] + 5313,09 + 0 \\ &= 23952,09 \text{ N} \end{aligned}$ 

When a motor frame connected to the rails are used, the weights connected to the rails effective on that side shall be taken into account. If the upwards braking is performed from the car,  $k_3$  coefficient shall be taken as equal to  $k_1$ . If the upwards braking is performed from the motor by electromechanical brake,  $k_3$  coefficient may be taken as equal to  $k_2$ . The upward braking mechanism is performed from the motor in this system. Motor frame is carried by three rails. It is assumed that the load is distributed equally between the rails. Half weight of the car and rated load, and the weight of the counterweight, ropes and motor act on the motor frame side with 3 rails.

 $\omega = 3,46$ 

 $k_{3.}M_{auxi} = [k_{3.}g_{n.}((P+Q)/2+G+M+H) / n]$ = 1.2\*9,81\*((800+1100)/2+1500+200+100)/3

 $\sigma_k = [(F_{v+} k_3.M_{aux}). \varpi] / A$ 

= (23952,09+10791)\*3,46/1725

$$\sigma_{\rm k}$$
 =69,687 N/mm<sup>2</sup>



If the upwards braking would be performed from the car, then 2 shall be used as the coefficient for  $k_3$  factor. In this case, the stress would be 84.117 N/mm<sup>2</sup>. However, this value is applicable for genuinely progressive safety gears. If a brake called as progressive, but that does not actually slide, it would be used 3 or 5 as the value of  $k_1$  factor, and thus the stress would be much greater. Then, the rail cross-section would not be adequate. This shall be considered as a great risk for the lift.

## c. Combined stress

 $\begin{array}{l} \text{Bending stress} \\ \sigma = \sigma_m = \, \sigma_x \, + \, \sigma_y \leq \sigma_{\text{perm}} \\ \sigma = \sigma_m = \, \sigma_x \, + \, \sigma_y = 29,398 \, \textit{N/mm}^2 \quad \leq \sigma_{\text{perm}} = 205 \, \textit{N/mm}^2 \end{array}$ 

Bending and compressive/tensile stresses

 $\begin{array}{ll} \sigma = \sigma_{m} + (F_{\nu} + k_{3}.M_{aux})/A & \leq \sigma_{perm} \\ = 29,398 + (23952,09 + 10791) \ /1725 \\ = 49,538 \ \textit{N/mm}^{2} \ \leq \sigma_{perm} = 205 \ \textit{N/mm}^{2} \end{array}$ 

Bending and buckling stress

 $σ = σ_k + 0.9 σ_m \le σ_{perm}$   $σ = 69,687 + 0.9^* 29,398$ 

 $\sigma$  = 96,145 *N/mm*<sup>2</sup>  $\leq \sigma_{perm}$  = 205 N/mm<sup>2</sup>

## d. Flange bending stress

 $h_1 = 75 \text{ mm}$  b = 20 mm/2 = 10 mm (half the width of the guide shoe lining) f = 10 mmc = 10 mm

L = 140 mm

For the mode of safety gear operation, the greatest  $F_x$  force is obtained in Case 1.

 $F_x = 691.159N$ 

As sliding type guide shoes are used in the system

$$\begin{split} \sigma_{F} = & (F_x^*(h_1\text{-}b\text{-}f)^*6)/(c^{2*}(L+2^*(h_1\text{-}f)) &\leq \sigma_{perm} \\ &= & (691,159^*(75\text{-}10\text{-}10)^*6)/(10^{2*}(140\text{+}2^*(75\text{-}10))) \\ &= & 8,447 \text{ N/mm}^2 &\leq \sigma_{perm} = 205 \text{ N/mm}^2 \end{split}$$

#### e. Deflections

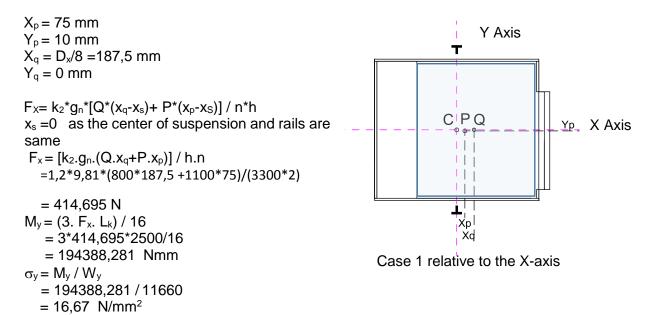
As the calculation of the deflection amounts is not related with the permissible stress value, they shall be performed once for the maximum  $F_X$  and  $F_Y$  values at the end of the calculations. The fact that the deflection is suitable for maximum forces allows to decide that it is adequate for smaller forces, too.

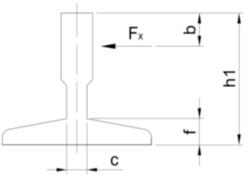
## 2. Normal operation, Running

For a material of 370 N/mm<sup>2</sup> under normal operation as per (2.25) Sf Safety factor  $\sigma_{perm} = 165$ .  $k_2 = 1.2$  impact factor

#### a. Bending stress

Case 1 Bending stress caused by the guiding forces related with the Y axis of the guide rail:

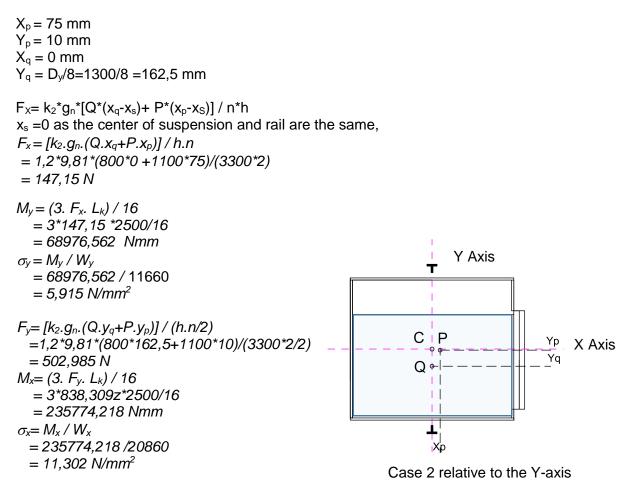




$$\begin{split} F_{Y} &= k_{2} \cdot g_{n} * [Q^{*}(y_{q} \cdot y_{s}) + P^{*}(y_{p} \cdot y_{s})] / (h^{*}n/2) \\ y_{s} &= 0 \text{ as the center of suspension and rails are same,} \\ F_{y} &= [k_{2} \cdot g_{n} \cdot (Q \cdot y_{q} + P \cdot y_{p})] / (h \cdot n/2) \\ &= 1,2^{*}9,81^{*}(800^{*}0 + 1100^{*}10) / (3300^{*}2/2) \\ &= 39,24 \text{ N} \\ M_{x} &= (3. F_{y}. L_{k}) / 16 \\ &= 3^{*}39,24^{*}2500/16 \\ &= 18393,75 \text{ Nmm} \\ \sigma_{x} &= M_{x} / W_{x} \\ &= 18393,75 / 20860 \\ &= 0,881 \text{ N/mm}^{2} \end{split}$$

Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y = 16,67+0,881$  $\sigma = \sigma_m = 17,551 \text{ N/mm}^2$ 

Case 2 Bending stress caused by the guiding forces related with the X axis of the rail:



Total bending stress  $\sigma = \sigma_m = \sigma_x + \sigma_y = 5,915+11,302$  $\sigma = \sigma_m = 17,217 \text{ N/mm}^2$ 

It shall be observed that the stress on Case 1 is higher when we compare total bending stresses. This stress shall be taken as the worse condition for the calculations.  $\sigma = \sigma_m = 17,551 \text{ N/mm}^2$ 

## **b. Buckling stress**

$$\begin{split} F_v &= (M_g^*g_n) + F_p \\ M_g^*g_n &= 13,54^*40^*9,81 = 5313,09 \ N \\ F_p &= 0 \\ F_v &= 5313,09 \\ k_3.M_{aux} &= [k_3.g_n.((P+Q)/2+G+M+H) \ / \ n] \\ &= 1.2^*9,81^*((800+1100)/2+1500+200+100)/3 \\ &= 10791 \ N \\ \sigma_k &= (F_{v+} k_3.M_{aux})/\ A \end{split}$$

= (5313,09+10791)/1725 = 9,3357 *N/mm*<sup>2</sup>

# c. Combined stress

Total bending stress

$$\begin{split} \sigma = &\sigma_m = \sigma_x + \sigma_y \ \leq \sigma_{perm} \\ = \ 16,67 + 0,881 \\ \sigma = &\sigma_m = \ 17,551 \ N/mm^2 \ \leq \sigma_{perm} = \ 165 \ N/mm^2 \end{split}$$

Bending and compressive/tensile stresses  $\sigma = \sigma_m + (F_v + k_3.M_{aux})/A = 17,551+9,3357 \le \sigma_{perm}$ 

 $= 26,886 \text{ N/mm}^2 \le \sigma_{\text{perm}} = 165 \text{ N/mm}^2$ 

d. Flange bending stress

 $h_1 = 75 \text{ mm}$  b = 20 mm/2 = 10 mm (half the width of the guide shoe lining) f = 10 mm c = 10 mmL = 140 mm

For the mode of the safety gear operation, the greatest  $\mathsf{F}_x$  force is obtained in Case 1.

 $F_x = 414.695 N$ 

As sliding type guide shoes are used in the system

$$\begin{split} \sigma_F &= (F_x^*(h_1 - b - f)^*6) / (c^2 * (L + 2^*(h_1 - f)) \leq \sigma_{perm} \\ &= (691, 159^*(75 - 10 - 10)^*6) / (10^{2*}(140 + 2^*(75 - 10))) \\ &= 5,068 \ \text{N/mm}^2 \ \leq \sigma_{perm} = 165 \ \text{N/mm}^2 \end{split}$$

## e. Deflection

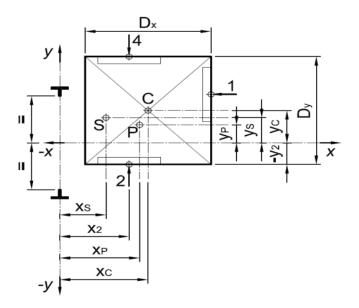
The calculation of the deflection shall be performed at the end of the operation and loading calculations after determining the maximum  $F_x$  value.

## 3. Normal operation, Loading

For a material of 370 N/mm<sup>2</sup> as per (2.25) Sf Safety factor  $\sigma_{\text{perm}}$  = 165.

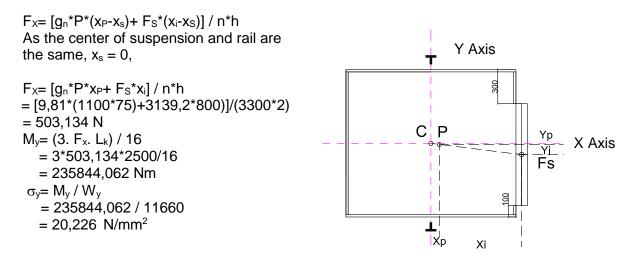
 $\begin{array}{l} F_{s} \mbox{ value - For passenger lifts:} \\ F_{s} = 0,4.g_{n} \ .Q \\ = 0,4^{*}9,81^{*}800 \\ = 3139,2 \ N \\ X_{i} = 1500/2 + 50 \ (\mbox{mid of the sill of the car}) \\ = 800 \ \mbox{mm} \end{array}$ 

Moment branches shall be taken as per the suspension center in the Normal operation, Loading mode. When the suspension center and rail center are different, the formulation provided here shall be used for the specific lift.



## a. Bending stress

Case 1 Bending stress caused by the guiding forces related with the Y axis of the guide rail:



Case 2 Bending stress caused by the guiding forces related with the X x axis of the guide rail:

$$\begin{split} F_{Y} &= [g_n^* P^*(y_P - y_s) + F_s^*(y_i - y_s] / (h^*n/2) \\ \text{As the center of suspension and rail are the same} \\ y_s &= 0, \\ \text{Due to the eccentricity of the door } Y_F = 200/2 = 100 \text{ mm} \\ F_Y &= [g_n^* P^* y_P + F_s^* y_i] / (h^*n/2) \\ &= [9,81^*(1100^*10) + 3139,2^*100)]/(3300^*2/2) \\ &= 127,82 \text{ N} \\ M_x &= (3. F_y. L_k) / 16 \\ &= 3^*127,82^*2500/16 \\ &= 59919,034 \text{ Nm} \\ \sigma_x &= M_x / W_x \\ &= 59919,034 / 20860 \\ &= 2,872 \text{ N/mm}^2 \end{split}$$

## **b. Buckling stress**

$$\begin{split} F_v &= (M_g^*g_n) + F_p \\ M_g^*g_n &= 13,54^*40^*9,81 = 5313,09 \ N \\ F_p &= 0 \\ F_v &= 5313,09 \\ k_3.M_{aux} &= [k_3.g_n.((P+Q)/2+G+M+H) \ / \ n] \\ &= 1.2^*9,81^*((800+1100)/2+1500+200+100)/3 \\ &= 10791 \ N \end{split}$$

$$\begin{split} \sigma_k &= (F_{v\,+}\,k_3.M_{aux})/\;A \\ &= (5313,09{+}10791)/1725 \\ &= 9,3357\;N/mm^2 \end{split}$$

## c. Combined stress

 $\begin{array}{l} \mbox{Total bending stress} \\ \sigma = \sigma_m = \ \sigma_x + \ \sigma_y = 20,226 + 2,872 \le \sigma_{perm} \\ = 23,098 \ \mbox{N/mm}^2 \ \le \ \sigma_{perm} = 165 \ \mbox{N/mm}^2 \ \end{array}$ 

 $\begin{array}{l} \text{Bending and compressive/tensile stresses,} \\ \sigma = \sigma_{m} + (F_{v} + k_{3}.M_{aux})/A \\ = 23,098 + 9,3357 \leq \sigma_{perm} \\ = 32,433 \ \text{N/mm}^{2} \leq \sigma_{perm} = 165 \ \text{N/mm}^{2} \end{array}$ 

## d. Flange bending stress

For normal operation, running and loading modes, the greatest  $F_x$  force is obtained in Case 1 of the Normal operation, running mode.

F<sub>x</sub> = 414,695 N

As sliding type guide shoes are used in the system  $\sigma_{F} = (F_{x}^{*}(h_{1}-b-f)^{*}6)/(c^{2*}(L+2^{*}(h_{1}-f)) \leq \sigma_{perm})$   $= (691,159^{*}(75-10-10)^{*}6)/(10^{2*}(140+2^{*}(75-10)))$   $= 5,068 \text{ N/mm}^{2} \leq \sigma_{perm} = 165 \text{ N/mm}^{2}$ 

## e. Deflections

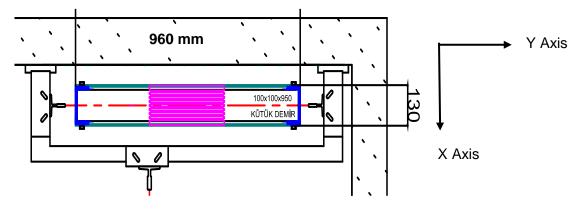
The greatests values shall be selected by taking the maximum  $F_x$  and  $F_y$  values obtained in the calculations for all operating modes into account. When the condition is examined, it is seen that the greatest forces occur in the operation mode of the safety gear.

 $\begin{array}{l} F_x = 691,159 \ N \ Safety \ gear \ operation \ Case \ 1 \\ F_y = 838,309 \ N \ Safety \ gear \ operation \ Case \ 2 \\ \\ \delta_y = (0,7 \ . \ F_Y \ . \ L^3) \ / \ (48 \ . \ E \ . \ l_x) + \ \delta \ _{str-y} \quad on \ the \ y-y \ plane \ \leq \ \delta_{perm} \\ = 0,7^* 838,309^* 2500^3) \ / \ (48^* 2,1^* 10^{5*} 1020000) \\ = 0,89 \ mm \ \leq \ \delta_{perm} = 5 \ mm \\ \\ \delta_x = (0,7 \ . \ F_X \ . \ L^3) \ / \ (48 \ . \ E \ . \ l_Y) \ + \ \delta \ _{str-x} \quad on \ the \ x-x \ plane \ \leq \ \delta_{perm} \\ = 0,7^* 691,159^* 2500^3) \ / \ (48^* 2,1^* 10^{5*} 524800) \\ = 1,429 \ mm \ \leq \ \delta_{perm} = 5 \ mm \end{array}$ 

# C) CALCULATION FOR RAILS FOR COUNTERWEIGHT OR BALANCING WEIGHT (Normal operation)

A mechanical brake is not used for the counterweight. In this case, calculations for the operation of the safety gear shall not be performed. Suspension point shall be taken as the axis of the moment branch for the calculations, and the calculations shall be performed as per this condition. Calculations for (normal operation, loading) are not performed for the counterweight calculations.

Counterweight dimensions, G = 1500 Kg,  $G_x = 130 \text{ mm}$ ,  $G_y = 960 \text{ mm}$   $x_G = \%10G_x = 13 \text{ mm}$ ,  $y_G = \%5G_y = 48 \text{ mm}$   $x_S = 0 \text{ mm}$ Suspension length = 3000 mm



#### **Bending stress**

 $F_{X} = [k_{2}.g_{n}.G. (x_{G}+x_{S})] / n.h$ = 1,2\*9,81\*1500\*13)/( 3000\*2) = 38,259 N M<sub>Y</sub> = 3.F<sub>X</sub>.L /16 = 3\*38,259\*2500/16 = 17933,91 Nm  $\sigma_{Y} = M_{Y} / W_{Y} = 17933,91 /5389$ = 3,33 N/mm<sup>2</sup>  $F_{Y} = (k_{2}.g_{n}.G. y_{G}) 2 / n.h$ = 1,2\*9,81\*1500\*48\*2/(3000\*2) = 282,528 N  $M_{X} = 3.F_{Y}.L /16$ = 3\*282,528\*2500/16 = 132435 Nm  $\sigma_{X} = M_{Y} / W_{Y} = 132435 /9169$ = 14.45 N/mm<sup>2</sup>

#### **Buckling stress**

 $F_{v} = (M_{g}^{*}g_{n}) + F_{p}$ = 40\*7,379\*9,81+0 = 2895,519 N k\_{3.}M\_{aux} = [k\_{3.}g\_{n.}((P+Q)/2+G+M+H) / n] = 1.2\*9,81\*((800+1100)/2+1500+200+100)/3

= 10791 N

 $\sigma_k = (F_{v+} k_3.M_{aux})/A = (2895,519+10791)/940$ 

= 14,560 N/mm<sup>2</sup>

#### Bending and compressive stresses

Total bending stress

$$\begin{split} \sigma = &\sigma_{m} = \sigma_{x} + \sigma_{y} &\leq \sigma_{perm} \\ &= 3,33 + 14,45 \\ &= 17,78 \; \text{N/mm}^{2} \; \leq \sigma_{perm} = \! 165 \; \text{N/mm}^{2} \end{split}$$

Bending and compressive/tensile stresses

$$\begin{split} \sigma &= \sigma_{M} + (F_{v+} k_{3}.M_{yardimci}) / \ A < &\sigma_{perm} \\ &= 17,78 + 14,560 \\ &= 32,34 \ N/mm^{2} \ \leq &\sigma_{perm} = 165 \ N/mm^{2} \end{split}$$

#### Flange bending stress

For sliding type guide blades  $\begin{aligned} \sigma_{F} &= (F_{x}^{*}(h_{1}\text{-}b\text{-}f)^{*}6)/(c^{2*}(L+2^{*}(h_{1}\text{-}f)) \leq \sigma_{perm} \\ &= 38,259^{*}(65\text{-}10\text{-}8)^{*}6/(6^{2*}(140\text{+}2^{*}(65\text{-}8))) \\ &= 1,17 \text{ N/mm}^{2} \leq \sigma_{perm} = 165 \text{ N/mm}^{2} \end{aligned}$ 

#### Deflections

 $\begin{array}{l} \delta_y = (0,7 \;.\; F_Y \,.\; L^3) \;/\; (48 \;.\; E \;.\; I_X) + \; \delta_{\; str \cdot y} & y \text{-y düzleminde} \; \leq \; \delta_{perm} = \; 10 mm \\ = \; (0,7^* 282,528^* 2500^3) / (48^* 2.1^* 10^{5*} 409500) \\ = \; 0,74 \; mm \leq \; \delta_{perm} = \; 10 mm \end{array}$ 

 $\begin{array}{ll} \delta_x = \left(0,7 \, . \, F_x \, . \, L^3\right) / \, (48 \, . \, E \, . \, I_Y) + \! \delta_{\, str \cdot x} & x \! - \! x \, d\ddot{u} z leminde & \leq \delta_{perm} \, = \, 10 \, \, mm \\ = \left(0,7^* 38,259^* 2500^3\right) / (48^* 2.1^* 10^{5*} 188600) \\ = 0,22 \, \, mm \leq \delta_{perm} \, = \, 10 \, mm \end{array}$ 

#### RAILS SELECTED ARE SUITABLE

#### DISCLAIMER

This manual is intended as a means for providing help for guidance of lifts and selection of guide rails within the cover of TS EN 81-20 and TS EN 81-50 standards. This manual is not intended for replacing the will of the lift technician designing the lift for their own examinations and assessments and making their own decisions. Asray and Serdar Tavasloğlu, as the compiler of this document, declares that they shall not accept any responsibility for measures that are taken or not taken based on this manual.

# "Omega" value $\varpi$ related to $\lambda$

λ	0	1	2	3	4	5	6	7	8	9	λ
20	1,04	1,04	1,04	1,05	1,05	1,06	1,06	1,06	1,07	1,08	20 .
30	1,08	1,09	1,09	1,10	1,10	1,11	1,11	1,12	1,13	1,13	30
40	1,14	1,14	1,15	1,16	1,16	1,17	1,18	1,19	1,19	1,20	40
50	1,21	1,22	1,23	1,23	1,24	1,25	1,26	1,27	1,28	1,29	50
60	1,30	1,31	1,32	1,33	1,34	1,35	1,36	1,37	1,39	1,40	60
70	1,41	1,42	1,44	1,45	1,46	1,48	1,49	1,50	1,52	1,53	70
80	1,55	1,56	1,58	1,59	1,61	1,62	1,64	1,66	1,68	1,69	80
90	1,71	1,73	1,74	1,76	1,78	1,80	1,82	1,84	1,86	1,88	90
100	1,90	1,92	1,94	1,96	1,98	2,00	2,02	2,05	2,07	2,09	100
110	2,11	2,14	2,16	2,18	2,21	2,23	2,27	2,31	2,35	2,39	110
120	2,43	2,47	2,51	2,55	2,60	2,64	2,68	2,72	2,77	2,81	120
130	2,85	2,90	2,94	2,99	3,03	3,08	3,12	3,17	3,22	3,26	130
140	3,31	3,36	3,41	3,45	3,50	3,55	3,60	3,65	3,70	3,75	140
150	3,80	3,85	3,90	3,95	4,00	4,06	4,11	4,16	4,22	4,27	150
160	4,32	4,38	4,43	4,49	4,54	4,60	4,65	4,71	4,77	4,82	160
170	4,88	4,94	5,00	5,05	5,11	5,17	5,23	5,29	5,35	5,41	170
180	5,47	5,53	5,59	5,66	5,72	5,78	5,84	5,91	5,97	6,03	180
190	6,10	6,16	6,23	6,29	6,36	6,42	6,49	6,55	6,62	6,69	190
200	6,75	6,82	6,89	6,96	7,03	7,10	7,17	7,24	7,31	7,38	200
210	7,45	7,52	7,59	7,66	7,73	7,81	7,88	7,95	8,03	8,10	210
220	8,17	8,25	8,32	8,40	8,47	8,55	8,63	8,70	8,78	8,86	220
230	8,93	9,01	9,09	9,17	9,25	9,33	9,41	9,49	9,57	9,65	230
240	9,73	9,81	9,89	9,97	10,05	10,14	10,22	10,30	10,39	10,47	240
250	10,55										

"Omega" value  $\varpi$  related to  $\lambda\,$  for steel with tensile stress of 370 N/mm2  $\,$ 

"Omega" value  $\varpi$  related to  $\lambda\,$  for steel with tensile stress of 520 N/mm2  $\,$ 

λ	0	1	2	3	4	5	6	7	8	9	λ
20	1,06	1,06	1,07	1,07	1,08	1,08	1,09	1,09	1,10	1,11	20
30	1,11	1,12	1,12	1,13	1,14	1,15	1,15	1,16	1,17	1,18	30
40	1,19	1,19	1,20	1,21	1,22	1,23	1,24	1,25	1,26	1,27	40
50	1,28	1,30	1,31	1,32	1,33	1,35	1,36	1,37	1,39	1,40	50
60	1,41	1,43	1,44	1,46	1,48	1,49	1,51	1,53	1,54	1,56	60
70	1,58	1,60	1,62	1,64	1,66	1,68	1,70	1,72	1,74	1,77	70
80	1,79	1,81	1,83	1,86	1,88	1,91	1,93	1,95	1,98	2,01	80
90	2,05	2,10	2,14	2,19	2,24	2,29	2,33	2,38	2,43	2,48	90
100	2,53	2,58	2,64	2,69	2,74	2,79	2,85	2,90	2,95	3,01	100
110	3,06	3,12	3,18	3,23	3,29	3,35	3,41	3,47	3,53	3,59	110
120	3,65	3,71	3,77	3,83	3,89	3,96	4,02	4,09	4,15	4,22	120
130	4,28	4,35	4,41	4,48	4,55	4,62	4,69	4,75	4,82	4,89	130
140	4,96	5,04	5,11	5,18	5,25	5,33	5,40	5,47	5,55	5,62	140
150	5,70	5,78	5,85	5,93	6,01	6,09	6,16	6,24	6,32	6,40	150
160	6,48	6,57	6,65	6,73	6,81	6,90	6,98	7,06	7,15	7,23	160
170	7,32	7,41	7,49	7,58	7,67	7,76	7,85	7,94	8,03	8,12	170
180	8,21	8,30	8,39	8,48	8,58	8,67	8,76	8,86	8,95	9,05	180
190	9,14	9,24	9,34	9,44	9,53	9,63	9,73	9,83	9,93	10,03	190
200	10,13	10,23	10,34	10,44	10,54	10,65	10,75	10,85	10,96	11,06	200
210	11,17	11,28	11,38	11,49	11,60	11,71	11,82	11,93	12,04	12,15	210
220	12,26-	12,37	12,48	12,60	12,71	12,82	12,94	13,05	13,17	13,28	220
230	13,40	13,52	13,63	13,75	13,87	13,99	14,11	14,23	14,35	14,47	230
240	14,59	14,71	14,83	14,96	15,08	15,20	15,33	15,45	15,58	15,71	240
250	15,83										